

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- III EXAMINATION – SUMMER 2015

Subject Code:130002

Date:06/06/2015

Subject Name: Advanced Engineering Mathematics

Time: 02.30pm-05.30pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) Solve $[1 + e^{x/y}]dx + e^{x/y}[1 - \frac{x}{y}]dy = 0$ 04
- (ii) Solve $\frac{dy}{dx} + \frac{y}{x^2} = 6e^{1/x}$ 03
- (b) Attempt the following
- (i) Solve $(D^3 - 7D + 6)y = e^{2x}$ 04
- (ii) Define square wave function and draw its graph 03
- Q.2** (a) Attempt the following
- (i) Solve $(D^2 + 9)y = \cos 2x + \sin 2x$ 04
- (ii) Find the ordinary and singular points of $2x^2y'' + 6xy' + (x+3)y = 0$ 03
- (b) Attempt the following
- (i) Solve the Cauchy-Euler equation $x^2D^2y - 3xDy + 5y = x^2 \sin(\log x)$ 05
- (ii) Define Gamma Function and obtain its value for 7. 02
- OR**
- (b) Find the series solution of $(1 + x^2)y'' + xy' - 9y = 0$ 07
- Q.3** (a) Find Fourier series for $f(x) = \begin{cases} \pi x, 0 \leq x \leq 1 \\ \pi(2-x), 1 \leq x \leq 2 \end{cases}$ 07
- (b) Attempt the following
- (i) Find Fourier series expansion of $f(x) = |x|, -\pi < x < \pi$ 04
- (ii) Find Fourier sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$. 03
- OR**
- Q.3** (a) Attempt the following
- (i) Obtain Fourier series for $f(x) = x - x^2, -1 < x < 1$. 04
- (ii) Find a cosine series for $f(x) = e^x, 0 < x < \pi$. 03
- (b) Obtain Fourier series to represent $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$. 07
- Q.4** (a) Attempt the following
- (i) Find the inverse Laplace transform of $\frac{4s+5}{(s-1)^2(s+2)}$ 04
- (ii) Find the Laplace transform of $e^{4t} \sin 2t \cos t$ 03
- (b) Attempt the following
- (i) Solve by Laplace transform $y'' + 6y = 1, y(0) = 2, y'(0) = 0$. 05

(ii) Find the convolution of $1*1$ 02

OR

Q.4 (a) Attempt the following

(i) Find the inverse Laplace transform of $\frac{2-5s}{(s-6)(s^2+1)}$ 04

(ii) Find Laplace transform of $t^2 \cosh 3t$. 03

(b) Attempt the following

(i) Solve by Laplace transform $y' - 4y = 2e^{2t} + e^{4t}$ given that at $t = 0, y = 0$. 05

(ii) Find Laplace transform of $(t-1)^2 u(t-1)$. 02

Q.5 (a) Attempt the following

(i) Derive partial differential equation by eliminating constants a and b from $z = (x+a)(y+b)$. 03

(ii) Solve by separation of variables method: $u_x + u_y = 2(x+y)u$. 04

(b) Use Frobenius method to solve $2x^2 y'' - xy' + (1-x^2)y = 0$. 07

OR

Q.5 (a) Attempt the following

(i) Form the partial differential equation by eliminating the arbitrary functions f and F from the relation $y = f(x-at) + F(x+at)$. 03

(ii) Find the complete integral of $pq = 4z$. 04

(b) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| \geq 1 \end{cases}$ as a Fourier integral. Hence evaluate 07

$$\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda .$$
