Seat No.: ________ **Enrolment No.___________**

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) - EXAMINATION – SUMMER 2017

Subject Code: 2130002 **Date: 25/05/2017**

Subject Name: Advanced Engineering Mathematics

Time: 10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	What are the order and the degree of the differential equation $y''+3y^2 = 3\cos x$.	
	2	What is the integrating factor of the linear differential equation: $y'-(1/x)y = x^2$	
	3	Is the differential equation $ye^x dx + (2y + e^x)dy = 0$ is exact? Justify.	
	4	Solve: $y''+11y'+10y = 0$.	
	5	Find particular integral of : $y'''+y'= e^{2x}$	
	6	If $y = (c_1 + c_2 x)e^x$ is a complementary function of a second order differential equation, find the Wronskian $W(y_1, y_2)$.	
	7	Find the value of $\Gamma\left(\frac{7}{2}\right)$	
	8	What is the value of the Fourier coefficients a_0 and b_n for $f(x) = x^2, -1 < x < 1$.	
	9	Find $L\{e^{3t+3}\}$	
	10	Find $L^{-1}\left(\frac{4}{s^2} - \frac{1}{(s^2+9)}\right)$	
	11	Find the singular point of the differential equation $(1-x^2)y''-2xy'+n(n+1)y = 0$	
	12	Obtain the general integral of $\frac{\partial^3 z}{\partial x^3} = 0$	
	13	Obtain the general integral of $p + q = z$	
	14	State the relationship between beta and gamma function.	
Q.2	**(a)**	Solve: $(x^2 + y^2 + 3)dx - 2xydy = 0$	**03**
	(b)	Solve: $\frac{dy}{dx} + (\tan x)y = \sin 2x, y(0) = 0$	**04**
	(c)	$(D^4 - 16)y = e^{2x} + x^4$, where $D \equiv d/dx$	**07**
		OR	
	(c)	Use the method of variation of parameters to find the	**07**

general solution of $y''-4y'+4y=\frac{e^{2x}}{x}$

Q.3 **(a)** Find half range sine series of $f(x)=x^3, 0\le x\le \pi$ **03**

(b) Find the Fourier integral representation of the function **04**

$$f(x)=\begin{cases}2, & |x|<2\\ 0, & |x|>2\end{cases}$$

(c) Find the Fourier series expansion for the 2π - periodic function $f(x)=x-x^2$ in the interval $-\pi\le x\le \pi$ and show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$ **07**

OR

Q.3 **(a)** Discuss about ordinary point, singular point, regular singular point and irregular singular point for the differential equation: $x^3(x-1)y''+3(x-1)y'+7xy=0$ **03**

(b) Use the method of undetermined coefficients to solve the differential equation $y''+9y=2x^2$ **04**

(c) Find the series solution of $(x^2+1)y''+xy'-xy=0$ about $x_0=0$. **07**

Q.4 **(a)** Solve: $(D^2-1)y=xe^x, \text{where } D\equiv d/dx$ **03**

(b) Solve: $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\sin(\ln x)$ **04**

(c) Use Laplace Transform to solve the following initial value problem: **07**

$$y''-3y'+2y=12e^{-2t}, y(0)=2, y'(0)=6$$

OR

Q.4 **(a)** Obtain $L\left\{e^{2t}\sin^2 t\right\}$ **03**

(b) Find $L^{-1}\left[\frac{s+7}{s^2+8s+25}\right]$ **04**

(c) Using Convolution theorem, obtain $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$ **07**

Q.5 **(a)** Find the Laplace Transform of $t\,e^{4t}\cos 2t$ **03**

(b) Form the partial differential equation from the following: **04**

1) $z=ax+by+ct$ 2) $z=f\left(\frac{x}{y}\right)$

(c) Using the method of separation of variables solve, $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$ where $u(x,0)=6e^{-3x}$ **07**

OR

Q.5 **(a)** Obtain the solution of the partial differential equation: $p^2-q^2=x-y$, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ **03**

(b) Solve: $y^2p-xyq=x(z-2y)$, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ **04**

(c) Find the solution of the wave equation **07**

$u_{tt} = c^2 u_{xx}, \ 0 \le x \le L$ satisfying the conditions:

$u(0,t) = u(L,t) = 0, \ u_t(x,0) = 0, \ u(x,0) = \frac{\pi x}{L}, 0 \le x \le L$

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